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SHORTER COMMUNICATIONS

HEAT TRANSFER IN MHD FLOW WITH ALIGNED FIELD ON A FLAT PLATE AT HIGH PRANDTL NUMBER

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NOMENCLATURE

a, defined as $\equiv f''(0)$;
b, defined as $\equiv h'(0)$;
C_h, coefficient of heat transfer;
e, magnetic Prandtl number defined as $\equiv R_m/R$;
E, defined as $\equiv U_\infty^2/C_pT_\infty$;
$f(\eta)$, dimensionless velocity along the plate defined as $\equiv (R/2x)^{\frac{1}{2}}\,\psi(x, y)$;
$h(\eta)$, dimensionless magnetic field intensity along the plate defined as $\equiv (R/2x)^{\frac{1}{2}}\,\psi_m(x, y)$;
H_∞, free-stream magnetic field intensity;
k, thermal conductivity;
K, defined as $\equiv f''^2 + Se^{-1}h''^2$;
l, characteristic length of body in flow;
$L(\ )$, differential operator defined as $\equiv d^2/dz + (az^2/2)\,d/dz$;
r, recovery factor;
P, Prandtl number defined as $\equiv \mu C_p/k$;
R, Reynolds number defined as $\rho U_\infty l/\mu$;
R_x, local Reynolds number $= \rho U_\infty x/\mu$;
R_m, magnetic Reynolds number defined as $= U_\infty l\mu_e\sigma$;
S, magnetic pressure number defined as $\mu_e H_\infty^2/\rho U_\infty^2$;
T, dimensionless temperature;
U_∞, free stream velocity;
x, y, dimensionless coordinates along the plate measured from the leading edge and normal to it;
z, outer variable defined by $\eta P^{\frac{1}{3}}$.

Greek symbols
η, similarity variable defined as $\equiv y(R/2x)^{\frac{1}{2}}$;
μ, fluid viscosity;
μ_e, magnetic permeability;
ρ, fluid density;
ψ, ψ_m, fluid and magnetic stream function;
σ, electrical conductivity.

Subscripts
w, wall;
r, recovery;
∞, free stream.

Superscripts
$'$, $''$, $'''$, first, second and third derivatives with respect to η.

1. INTRODUCTION

HEAT transfer in the boundary-layer flow of an electrically conducting, incompressible fluid with aligned magnetic field and no pressure gradient is studied at high Prandtl number P, by the method of matched asymptotic expansions. The problem of high P is of interest in liquids and vapours which generally have a small value of electrical conductivity. Thus the magnetic Prandtl number e is assumed to be of order one or less. The analysis is later shown to give accurate results even if P is of order unity. The problem has been studied by various workers [1–4] for P of order unity. The governing equations for the present problem in standard notation [1, 4]† are

$$f''' + ff'' - Shh'' = 0, \tag{1a}$$

$$h'' + e(fh' - f'h) = 0, \tag{1b}$$

$$P^{-1}T'' + fT' + E(f''^2 + Se^{-1}h''^2) = 0 \tag{1c}$$

and the corresponding boundary conditions are

$$f(0) = 0 = f'(0), \qquad f'(\infty) = 1, \tag{2a}$$

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† The boundary conditions $f'(\infty)$ and $h'(\infty)$ are normalized to unity.

$$h(0) = 0, \qquad h'(\infty) = 1, \tag{2b}$$

$$T(0) = T_w \quad \text{or} \quad T'(0) = 0, \qquad T(\infty) = 1. \tag{2c}$$

The equations (1a, 1b) with (2a, 2b) are the well known equations of Greenspan and Carrier [6] (cited as GC below), whose solutions have been studied extensively in the past. We shall be using later the following solution of the GC equations for small η,

$$f(\eta) = a\eta^2/2 - a^2\eta^5/5! + O(\eta^6) \tag{3a}$$

$$h(\eta) = b\eta + eab\eta^4/4! + O(\eta^7) \tag{3b}$$

where $a = f''(0)$ and $b = h'(0)$ are functions of S and e and may be obtained from [6–8].

2. SOLUTION AT HIGH *P*

If $P \gg 1$ (i.e. small thermal conductivity), then the momentum boundary layer is much thicker than the thermal boundary layer. In general such a fluid has a small e, hence the velocity boundary layer is contained in the magnetic boundary layer. For the present case (in the terminology of the method of matched asymptotic expansions) we need only two expansions, the outer and the inner, as the thermal layer is contained in both the velocity and the magnetic layers (for $P \to \infty$, this is so when $P/e \to \infty$, i.e. P approaches infinity faster than e). Thus the present analysis certainly holds when e is low or of order unity. We now study the energy equation (1c) with boundary conditions (2c).

Outer solution. First we define an outer limit as the process $P \to \infty$ with η fixed; and writing

$$T = \sum_{n=0} T_n P^{-n/3} \tag{4}$$

in (1c), the coefficients of like powers of P give

$$fT'_m = -T''_{m-3} - EK\delta_{0m}, \qquad m = 0, 1, 2, \ldots. \tag{5}$$

Here $K = f''^2 + Se^{-1}h''^2$, δ_{ij} is the Kronecker delta and $T_{-1} = T_{-2} = T_{-3} = 0$. The above solutions do not satisfy the boundary conditions at the wall and are singular there. The inner expansion of the outer solution (5) is

$$T = 2aE/\eta + 1 + E\,J(0, S, e) + \ldots + O(P^{-1}\ln\eta) \tag{6}$$

where

$$J(\eta, S, e) = \int_{\eta}^{\infty} K/f - 2a/\eta^2 \, d\eta$$

is regular near the wall.

Inner solution. From an order of magnitude analysis we introduce the following inner variables

$$z = \eta P^{\frac{1}{3}}, \qquad t = TP^{-\frac{1}{3}} \tag{7}$$

and study the limit $P \to \infty$ with z and t fixed. Inspection of (6) and (7) suggests the following inner expansion

$$t = \sum_{n=0}^{3} t_n P^{-n/3} + O(P^{-\frac{4}{3}}\ln \mathrm{P}). \tag{8}$$

Substituting (3), (7), (8) in (1c) we get

$$L(t_0) = -Ea^2, \qquad L(t_1) = 0 = L(t_2), \tag{9a, b, c}$$

$$L(t_3) = a^2z^5t_{0z}/5 + Ea^3z^3/3 \tag{9d}$$

where $L(\text{operator}) = d^2/dz^2 + (az^2/2)\,d/dz$.

Results. For an insulated wall case, matching the solutions of the inner and outer equations gives the wall recovery factor

$$r = 2(T_r - 1)/E = 5{\cdot}266(a^4P)^{\frac{1}{3}} + 2J(0, S, e) + 3{\cdot}511(a^4/P)^{\frac{1}{3}} + O(P^{-1}\ln P). \tag{10}$$

For a nonmagnetic case ($S = 0$) the above result becomes

$$r = 1{\cdot}922\,P^{\frac{1}{3}} - 1{\cdot}341 + 0{\cdot}468\,P^{-\frac{1}{3}} + \ldots \tag{11}$$

In (11) the first two terms are due to [5].

For the heat transfer problem, matching of inner and outer solutions gives the heat transfer coefficient C_h at the wall as,

$$C_h\sqrt{(2R_x)} = P^{-1}T'(0)/(T_r - T_w) \tag{12a}$$

$$= \frac{(36a)^{\frac{1}{3}}}{2\Gamma(\frac{1}{3})}\left[P^{-\frac{2}{3}} - P^{-\frac{5}{3}}/45 + O(P^{-2})\right] \tag{12}$$

where R_x is the local Reynolds number.

3. DISCUSSION

The recovery factor result (10) obtained here is compared in Fig. 1 with previous calculations. The integral $J(0, S, e)$ in (10) is evaluated from Ramamoorthy's [7] numerical solution of the GC equations (which is only for $e = 0{\cdot}6$ and $0 < S \leqslant 0{\cdot}6$) by Simpson's rule. It is seen that our result for the nonmagnetic case at $P = 1$ gives $r = 1{\cdot}049$, which is about 5 per cent above Pohlhausen's [9] value and at $P = 15$ the present result underestimates by 2 per cent while the large P result of Meksyn [11] overestimates by about 34 per cent, Narasimha and Vasantha [5] underestimates by 4 per cent. Further, even with magnetic field

Table 1. Comparison of C_h *with previous work for* $e = 0{\cdot}6$ *and* $P = 1$

S	Present result, equation (12)	Exact results [1]
0·0	0·4684	0·4696
0·1	0·4588	0·4599
0·3	0·4359	0·4381
0·5	0·4051	0·4079

our result is in good agreement with the numerical result of [1]. Our result (12), for the coefficient of heat transfer is compared in Table 1 with the numerical results of [1] (with proper interpretation, see, Loeffler [10]) and [9]. For $P = 1$ our results underestimate by 0·25 per cent when compared

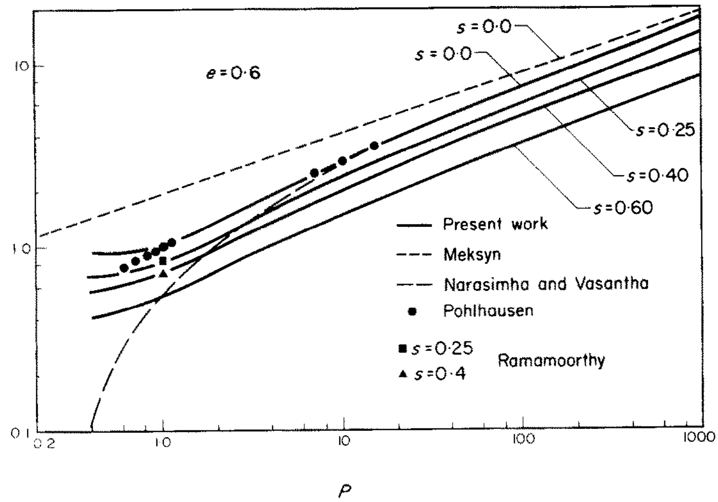

FIG. 1. Comparison of present work with previous results for recovery factor.

with [9] and at the most 0·5 per cent with [1]. For $P = 0{\cdot}6$, it is 0·4 per cent below the Pohlhausen's value. Moreover, for given P and e, r and C_h both decrease as S increases owing to the deceleration of the fluid by the induced Lorentz force in the boundary layer. This feature has also been observed by Ramamoorthy [1] and Ingham [3]. Qualitatively the variation of r and C_h with S and e is similar to that of skin friction a. On the other hand, if S and e are held fixed, r increases and C_h decreases as P increases.

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